

Prove $(\sec x \csc x - \tan x)(\sec x \csc x - \cot x) = 1$.

↓

$$\left(\frac{1}{\cos x} \frac{1}{\sin x} - \frac{\sin x}{\cos x} \right) \left(\frac{1}{\cos x} \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \left(\frac{1 - \sin^2 x}{\cos x \sin x} \right) \left(\frac{1 - \cos^2 x}{\cos x \sin x} \right)$$

$$= \frac{\cancel{\cos^2 x}}{\cancel{\cos x} \cancel{\sin x}} \frac{\cancel{\sin^2 x}}{\cancel{\cos x} \cancel{\sin x}}$$

$$= 1$$

$$\frac{1 - \sec^2 t}{1 - \cos^2 t}$$

$$= \frac{-\tan^2 t}{\sin^2 t}$$

$$= \frac{-\cancel{\sin^2 t}}{\cos^2 t} \cdot \frac{1}{\cancel{\sin^2 t}}$$

$$= -\sec^2 t$$

$$\frac{\csc \alpha - \sec \alpha}{\sin \alpha - \cos \alpha}$$

$$\frac{\csc \alpha - \sec \alpha}{\sin \alpha - \cos \alpha}$$

$$= \frac{\frac{1}{\sin \alpha} - \frac{1}{\cos \alpha}}{\sin \alpha - \cos \alpha} \cdot \frac{\sin \alpha \cos \alpha}{\sin \alpha \cos \alpha}$$

$$= \frac{\cos \alpha - \sin \alpha}{(\sin \alpha - \cos \alpha) \sin \alpha \cos \alpha}$$

$$= -\csc \alpha \sec \alpha$$

$$\cot^2 y + \tan^2 y = \csc^2 y \sec^2 y - 2$$

↓

$$= (1 + \cot^2 y)(1 + \tan^2 y) - 2$$

$$= 1 + \tan^2 y + \cot^2 y + \cot^2 y \tan^2 y - 2$$

$$= \cancel{1} + \tan^2 y + \cot^2 y + \frac{\cancel{1}}{\cancel{\tan^2 y}} \cancel{\tan^2 y} - \cancel{2}$$

$$= \tan^2 y + \cot^2 y$$

$$\frac{1 + \sin \theta}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = 2 \tan \theta$$

↓

$$= \frac{(1 + \sin \theta)^2 - \cos^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{1 + 2 \sin \theta + \sin^2 \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{2 \sin \theta + 2 \sin^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{2 \sin \theta \cancel{(1 + \sin \theta)}}{\cos \theta \cancel{(1 + \sin \theta)}} = 2 \tan \theta$$